Meet the Time Dependent Maxwell Equations

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The course consists of 3 lectures taking place in Room A.0.6, Ricamo Building (*Coppito 1*), on the following dates:

- **April 8**, Wednesday 2:30pm-4:30pm
- **April 9**, Thursday 2:30pm-4:30pm
- **April 10**, Friday 2:30pm-4:30pm.

Outline. Maxwell's Equations are a bit strange on first view. The goal is to understand them through simple examples that also highlight major scientific advances. Basic PDE and Real Analysis plus first year Physics is adequate background.

The electric field, magnetic field, and current density E, B, j are vector fields. The charge density is a scalar field ρ. They are related by the eight Maxwell equations plus conservation of charge.

• The equations are not independent.

• The laws that Maxwell inherited have the same static solutions as Maxwell's equations. For time dependent fields the premaxwell equations are inconsistent. When there are solutions, the premaxwell system have action at a distance, while Maxwell's equations have finite speed of propagation.

• The speed of propagation is expressed in terms of data from simple static measurements. It agrees with the observationally determined speed of light. We give three ways to derive the speed. Two are standard. The third generalizes D'Alembert's solution of the one dimensional wave equation.

• In vacuum, the fields E, B satisfy D'Alembert's wave equation, $u_{tt} - \Delta u = 0$. Maxwell used the retarded exact solution in terms of means on spheres of the initial data.

• There is a misconception that at all points and all solutions the fields E and B are orthogonal. The error deserves to be better known.

• Solutions of Maxwell's equations differ from general solutions of $u_{tt} - \Delta u = 0$. Two examples illustrate this point. The first considers $\int_0^\infty \, dt$ of solutions. For Maxwell this integral vanishes while for the D'Alembert's equation it need not. The second asks on how small a set can a solution of wavelength λ be concentrated. The intuitive answer $\sim 1/\lambda$ is right but the sharp constant for Maxwell is strictly larger by a factor $3/2$ than for solutions of the wave equation.